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A Systematic Approach to Improve BOC Power Spectrum for GNSS

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ABSTRACT

An analysis of digital Phase-modulated signals is performed based on frequency spectrum which consists of a continuous and a number of discrete components at multiples of clock frequencies. The analysis shows that these components depend on the pulse shape function of multi-level digital signals to be phase modulated. In this paper, the effect of duty cycle, rise and fall times of these multi-level digital signals, on the frequency spectrum is studied. It is observed that the duty cycle variation of 10% results 30 dB increase in undesired component and the 10% increase in rise & fall times increase the power of undesired component by 12 dB. The theoretical observations of the effects are applied on the Binary Offset Carrier (BOC) modulated signals as a case study, to discuss their effects in Global Navigation Satellite Systems (GNSS).

***Keywords* -** PSK, BOC, GNSS, Pulse duration, Rise time, Fall time

I. INTRODUCTION

In current digital technology era, the information is mostly transferred using digital phase-modulated signals [1], considering the ease in demodulator design and simple detection techniques using maximum-likelihood [2]. The digital phase-modulated signals are allowed to have multiple levels [3] instead of just two levels (0 and 1). These multi-level signals are assumed to be represented by independent non-overlapping pulses. These pulses can be characterized by different pulse shapes and with the different occurrence probabilities for correlation with spectrum of pulse shapes. The spectral contents other than desired in band are difficult to remove by filter of complex digital taps [4]. In this paper, we discuss a simplified solution by appropriate analysis of effects of pulse shape followed by demonstrating it on practical circuit with choices of appropriate devices. A number of technical literatures have been surveyed. Although only few papers discuss about the effect of a pulse shape on its power spectrum [3] but the effect of pulse shapes is not correlated with the modulated frequency spectrum. Here Section II explains the theory regarding the digital phase modulation, pulse shape, GNSS and BOC signals [5] and the importance of pulse shape in GNSS spectrum. Section III illustrates the simulation results with varying pulse shape parameters. Section IV describes the implementation of the theoretical observations in the practical circuit with concluding the paper in Section V.

II. THEORY

The digital phase modulated signal is represented by

$$s(t) = \cos(\omega_c t + \varphi(t)) \qquad (1)$$

ω_c in equation (1) defines the carrier frequency ($\omega_c = 2\pi f_c$) and $\varphi(t)$ shows the multi-level digital baseband-encoded signal given by

$$\varphi(\mathrm{t}) = \sum_{\substack{1\le \mathrm{r} \le \mathrm{n} \\ -\infty<k<\infty}} a_{kr}\, p_r(\mathrm{t}-\mathrm{kT}) \qquad (2)$$

$p_r(\mathrm{t})$ is the pulse shape of duration T, i.e.

$$p_r(\mathrm{t}) = 0;\quad \mathrm{t} < 0, t > T \qquad (3)$$
$$for \quad r = 1,2,3, \dots\dots\dots, n$$

All the pulse shapes for n- multi-level signals are assumed to be unique so equation (2) becomes

$$\varphi(\mathrm{t}) = \sum_{\substack{1\le \mathrm{r} \le \mathrm{n} \\ -\infty<k<\infty}} a_{kr}\, p(\mathrm{t}-\mathrm{kT}) \qquad (4)$$

a_{kr} is discrete variable, which can take any value out of n-levels at any time for duration T.

The variables a_{kr} are mutually exclusive as only one pulse signal can exist at a time.

The resultant multilevel digital phase-modulated signal using equation (4) in equation (1) is

$$s(t) = \cos[\omega_c t + \sum_{\substack{1\le \mathrm{r} \le \mathrm{n} \\ -\infty<k<\infty}} a_{kr}\, p(\mathrm{t}-\mathrm{kT}) \qquad (5)$$

Equation (5) represents the fundamental digital phase-modulated signal with uniform pulse shapes with same timing characteristics, for simplifying the spectral analysis. This equation shows that the frequency spectrum of $s(t)$ depends greatly on the pulse shape function $p(\mathrm{t})$. This adds up many discrete components in frequency spectrum at frequencies shifted from ω_c by multiple of the timing frequency (1/T) i.e. at $\left(f_c \pm \frac{k}{T}\right)$. Various factors of $p(t)$ like

(i) pulse duration ($t \leq T$);
(ii) rise time (t_r) and fall time (t_f)

contribute to the unwanted components in frequency spectrum which are discussed in section III.

The Global Navigation Satellite Systems (GNSS) using space-based navigation technology are using state-of-the-art Binary Offset Carrier (BOC) modulation [5], [6] based on the spread spectrum [7]. This is an advanced modulation that better shares the available frequency band among all the required signals by increasing spectral separation. In this way, the BOC modulation conserves the spectrum. This sharing of frequency band with information at different positions shifted from carrier frequency [8] facilitates the interoperability and compatibility of different navigation systems like GPS, Galileo, IRNSS etc. Hence the unwanted components due to pulse shape or otherwise are not allowed to be transmitted from on-board satellite and are necessary to avoid in modulated signal.

In this perspective, we carry theoretical analysis by simulations of pulse shape effects on to BOC modulation. It is observed that the duty cycle variation of 10% results 30 dB rises in undesired component and with the rise & fall time increase from 1% to 5% of pulse duration T, the power of undesired components increases by 7 dB.

III. SIMULATION

The multilevel digital modulated signal in equation (5) is used, with the uniform pulse signals having maximum duration (i.e. T) and zero rise-fall times, to simulate the ideal situation. Assuming equal probability of each level, the Fourier transform of equation (5) relative to positive frequencies is given by

$$S_r(\omega) = \frac{T}{2} e^{j\varphi_r} e^{\frac{j(\omega_c-\omega)T}{2}} \left[\frac{\sin\left(\frac{(\omega_c-\omega)T}{2}\right)}{\frac{(\omega_c-\omega)T}{2}}\right]^2 \qquad (6)$$

where φ_r is the phase deviation of r^th^ level out of n-pulses, those can be either positive or negative.

Therefore, the power spectrum of a multi-level (equidistant and equi-probable) phase modulated signal comes as

$$\mathrm{P}(\omega) = \frac{T}{4} \left[\frac{\sin\left(\frac{(\omega_c-\omega)T}{2}\right)}{\frac{(\omega_c-\omega)T}{2}}\right]^2 \qquad (7)$$

In view of its importance in GNSS signals, the BOC (5, 2) modulation technique is chosen for the further analysis. This modulation technique of BOC (5, 2) uses a subcarrier of 5.115 MHz for a spreaded signal of data rate 2.046 Mbps [9]. Both are multiplied to generate a BOC (5, 2) modulated baseband multi-level digital signal at data rate of 10.23 Mbps (=1/T) as shown in Fig 1.

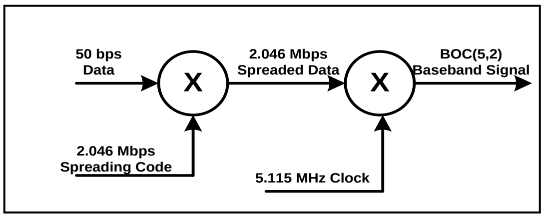

Fig.1. BOC (5, 2) Modulated Baseband Multi-Level Digital Signal

As the equation (6) represents the pulse frequency response, the power spectrum for BOC (5, 2) modulated signal at carrier frequency (ω_c) of 102.3 MHz is shown in Fig 2 based on equation (7). This power spectrum clearly indicates that in case of 50% duty cycle and zero rise-fall time, there appears no discrete or continuous component at frequency $\left(\omega_c \pm \frac{1}{T}\right)$[*]. Further analysis would consider only the first component (k=1), assuming that other components are out of the signal band and hence filtered out before transmission.

The effects of pulse shape parameters on the signal power spectrum are studied in the subsequent subsections.

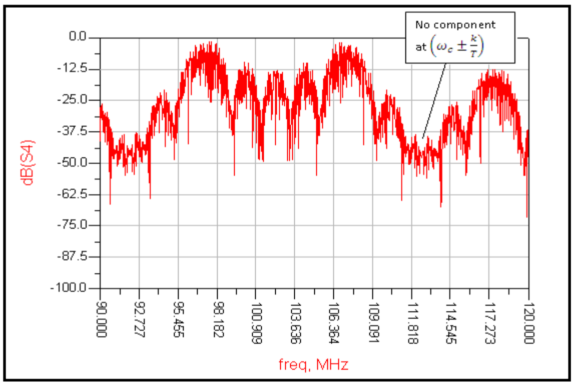

Fig.2. Power Spectrum at ω_c of 102.3 MHz for BOC (5, 2) Modulated Signal with Pulse Duration T and Zero Rise & Fall Times

A. *DUTY CYCLE*

Duty cycle of the used clock of 5.115 MHz is another way to represent the pulse width ($\tau \leq T$) of $p(t)$, where the pulse period is T.

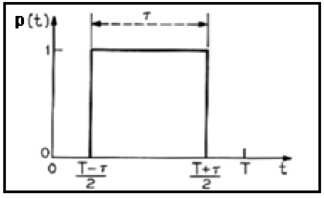

Fig.3. Pulse of Width $\tau \leq T$ used to Represent Multi-Level Digital Baseband Signal

[*] Hereafter ω_c is taken as frequency (Hz), but in standard it has unit of rad/sec.

Here in BOC (5, 2) modulated signal the multi-level digital baseband signals $\varphi(\mathrm{t})$ are represented by the pulses $p(\mathrm{t})$ of period T (=1/10.23 µsec) as shown in Fig 3. When the clock has a duty cycle other than 50%, a continuous frequency pattern appears at frequencies shifted from ω_c with multiples of $\frac{1}{\mathrm{T}}$. Fig. 5 shows the power spectrum of BOC (5, 2) modulated signal with clock of 48% duty cycle, which states clearly that the undesired continuous component has increased by 18 dB, compared to that of Fig. 4 with 50% duty cycle.

Here considering zero rise and fall times, the plot for integrated power of undesired continuous component of 2.046 MHz band centered at $\left(\omega_c \pm \frac{1}{T}\right)$ with variation in duty cycle around 50% is shown in Fig. 6.

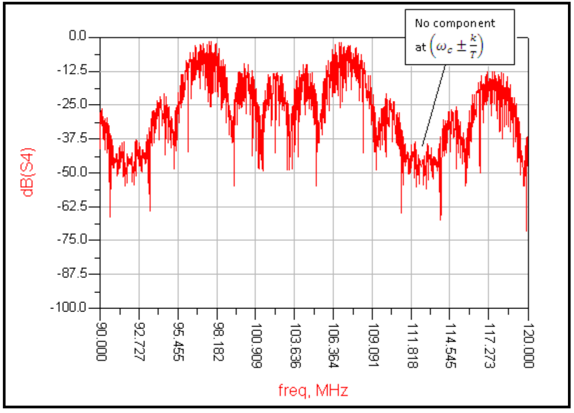

Fig.4. Power Spectrum at ω_c of 102.3 MHz for BOC(5, 2) Modulated Signal with Clock Duty Cycle 50% $(\tau = \mathrm{T})$ and Zero Rise & Fall Times

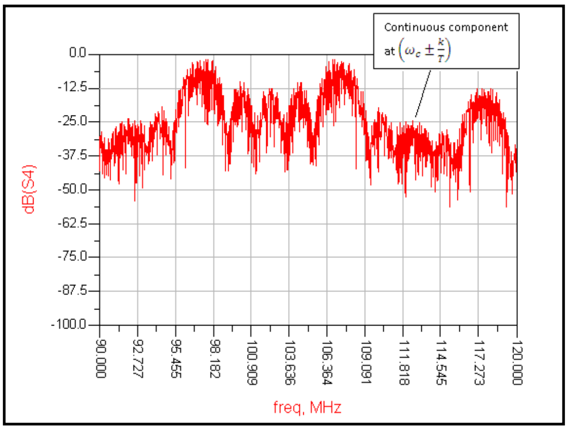

Fig.5. Power Spectrum at ω_c of 102.3 MHz for BOC (5, 2) Modulated Signal with Clock Duty Cycle 48% $(\tau < T)$ and Zero Rise & Fall Times

This quantifies that as duration of multi-level digital modulated pulses varies from 50% to 50±10%, the continuous spectrum component at frequency $\left(\omega_c \pm \frac{1}{T}\right)$ increases by 30 dB. The severity of this continuous spectral band can create noise for other omnipresent GNSS signals.

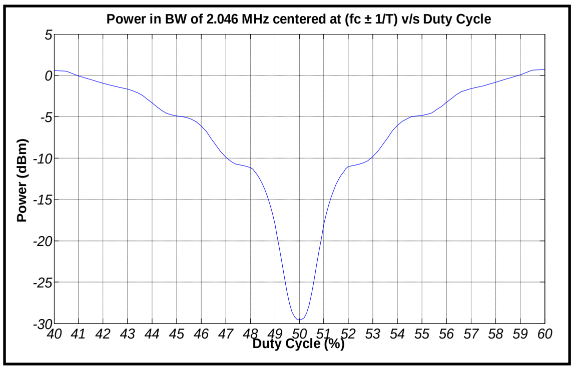

Fig.6. Power of undesired band centered at $\left(f_c \pm\right.$ *1T,* with ωc of 102.3 MHz for BOC (5, 2), varying with Clock Duty Cycle $(\tau \leq \mathrm{T})$ at Zero Rise & Fall Times

B. RISE & FALL TIMES

Finite rise & fall times (t_r & t_f) in the baseband multi-level digital modulated pulses, shown in Fig. 7, contribute to the discrete frequency components at frequencies $(\omega_c \pm \frac{k}{T})$.

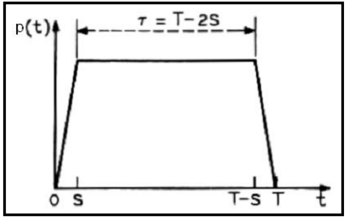

Fig.7. Pulse of width T and finite rise & fall times $(s < T)$ used to represent multi-level digital baseband signal

The localized power spectrum at f $i.e.\left(\omega_c \pm \frac{k}{T}\right)$ decreases as $(\frac{1}{\omega^4})$ as ω tends to infinite [3]. The simulation results of BOC (5, 2) modulated signal spectrum with pulses $p(\mathrm{t})$ of period T (=1/10.23 µsec) and finite t_r & t_f of 3 nsec, is shown in Fig. 8.

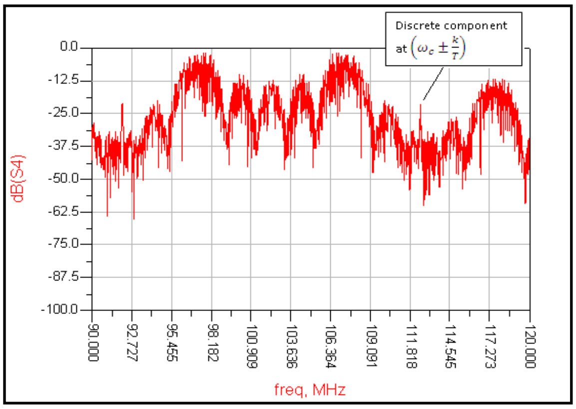

Fig.8. Power Spectrum at ω_c of 102.3 MHz for BOC(5, 2) Modulated Signal with Clock Duty Cycle 50% $(\tau = \mathrm{T})$ and 3 nsec Rise & Fall Times

The appearance of discrete component at $\left(\omega_c \pm \frac{1}{T}\right)$ in spectrum of Fig.8 is visible, increased by 20 dB compared to that of Fig. 2, but the integrated power in the undesired band around $\left(\omega_c \pm \frac{1}{T}\right)$ increases by 6 dB only. Hence a signal spectrum with undesired discrete components is more vulnerable than the undesired continuous (spreaded) component to affect other GNSS signals.

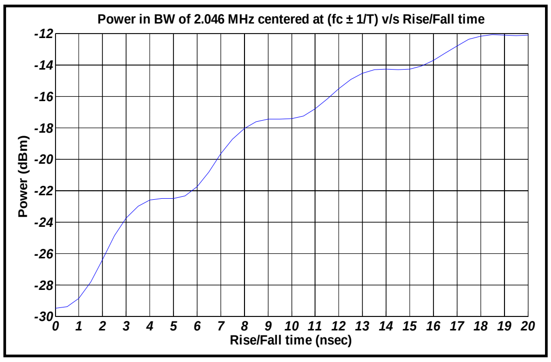

Fig.9. Power of undesired discrete component at $\left(\omega_c \pm \frac{1}{T}\right)$, with ω_c of 102.3 MHz for BOC (5, 2), varying with Rise & Fall Times at 50% Clock Duty Cycle $(\tau \leq \mathrm{T})$

Considering the 50% duty cycle, the plot of variation of the integrated power in the undesired band around $\left(\omega_c \pm \frac{1}{T}\right)$ is shown in Fig. 9 which indicates this power increase by 12 dB for t_r & t_f as 10% of T. Here it increases by around 7 dB when t_r & t_f are increased from 0 to 4 nsec.

C. COMBINED EFFECT

The combined effect of pulse width and rise & fall times is also studied here and the result, with duty cycle of 45% and rise-fall time of 3 nsec, is shown in Fig. 10. It indicates the increase in both the undesired continuous and discrete components around $\left(\omega_c \pm \frac{1}{T}\right)$.

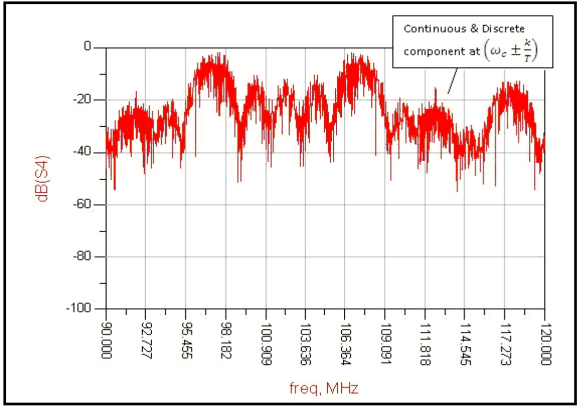

Fig.10. Power Spectrum at ω_c of 102.3 MHz for BOC (5, 2) Modulated Signal with Clock Duty Cycle 45% $(\tau < T)$ and 3 nsec Rise & Fall Times

The power of undesired component v/s duty cycle with different rise and fall times is shown in Fig 11. It highlights the importance of rise-fall time at duty cycle near to ideal i.e. 50%. But as the duty cycle deviates more from the ideal condition, the deviation in undesired power component due to rise-fall time, becomes continuously negligible.

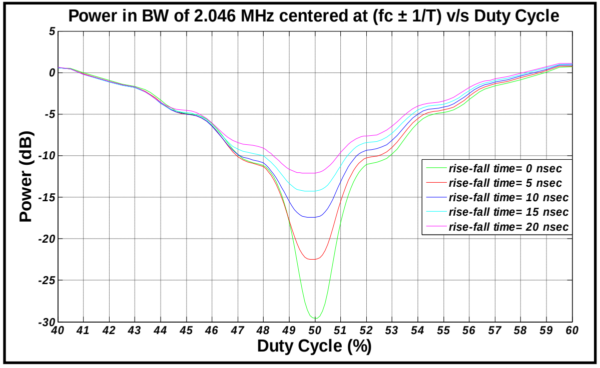

Fig.11. Power of undesired band centered at $\left(\omega_c \pm 1T,\right.$ with ωc of 102.3 MHz for BOC (5, 2), v/s Clock Duty Cycle $(\tau \leq \mathrm{T})$ at different Rise & Fall Times

IV. IMPLEMENTATION AND RESULTS

A BOC (5, 2) modulator hardware as per Fig. 1 has been developed and tested with devices of different timing parameters.

In hardware, the signal pulses are latched before carrier signal multiplication which brings the pulse width i.e. duty cycle of multilevel digital signals towards ideal, therefore the effect of duty cycle doesn't reflect in the implementation results. But the rise and fall time parameters for each used device are defined so the rise and fall time of multilevel digital pulses is the only factor to degrade the spectrum in real hardware.

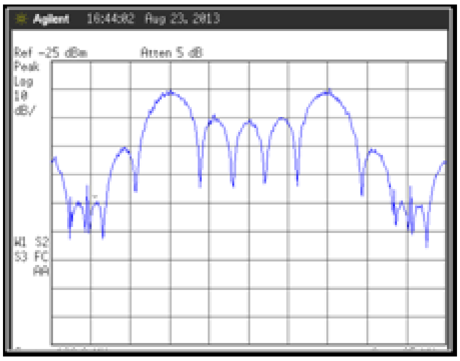

Fig.12. Power Spectrum at ω_c of 102.3 MHz for BOC (5, 2) Modulated Signal Designed with 74LS TTL Devices

(a) The BOC (5, 2) modulator was designed using the available slow speed 74 series TTL ICs of 'LS' class. The modulated power spectrum is shown in Fig. 12 and it clearly indicates that due to high rise and fall times, discrete components occur at frequency$\left(\omega_c \pm \frac{1}{T}\right)$.

(b) Later, to remove these discrete frequency components, a device of very low rise and fall time was used in design.

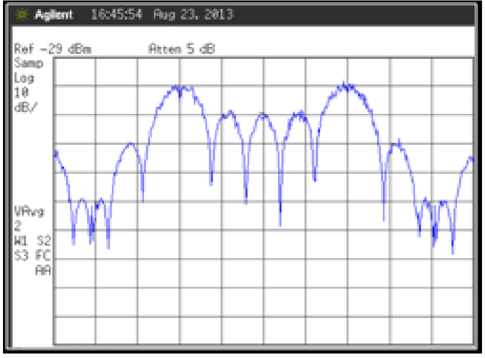

Fig.13. Power Spectrum at ω_c of 102.3 MHz for BOC (5, 2) Modulated Signal Designed using Fast Speed 74F TTL Devices

The high speed 74 series TTL ICs of ‘F’ class were used which have the rise and fall times (~6 nsec) one fourth than ‘LS’ class devices (~25 nsec). The improved spectrum is shown in Fig. 13 having tolerable discrete components at frequency$\Big(\omega_c \pm$ *1T*which has decreased by almost 10 dB compared to earlier configuration.

Hence, for a tolerable limit of 10 to 12 dB increase in power of undesired component, the duty cycle and rise-fall times can be deviated upto 1% and 5% respectively from the ideal condition. Above analysis of shape and timing characteristics of the multi-level digital modulated pulses shows that the pulse duration and rise & fall times play a significant role in the modulated spectrum which may be critical in GNSS applications.

V. CONCLUSION

In this paper, the study of the pulse shape and timing characteristics for a multi-level digital phase-modulated signal has been accomplished. Here the results, both of the simulated and developed modulators clearly describe the importance of the pulse shape in terms of duration and rise & fall times, for a modulator and especially for Global Navigation Satellite Systems (GNSS). So based on the above analysis to study and find out the tolerable limits of undesired power components, it’s very important to take the timing parameters, of devices to be used, in consideration while designing a multi-level digital phase modulator.

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